1 stepped pressure equilibrium code : sw03aa

Contents

1.1 outline

1. Given the Fourier harmonics of the interface geometry, return spectral constraints.

1.1.1 spectral width

2. The geometry of an interface is described by two functions, $R = \sum_j R_j \cos(m_j\theta - n_j\zeta)$ and $Z = \sum_j Z_j \sin(m_j\theta - n_j\zeta)$. (See `global` and `co01a` for more details.)
3. The spectral width is defined

$$M = \frac{1}{2}\sum_j (m_j^p + n_j^q)\left(R_j^2 + Z_j^2\right). \tag{1}$$

where $p \equiv$ `pwidth`, $q \equiv$ `qwidth` are positive integers given on input, and $m_j^p = 0$ for $m_j = 0$, $n_j^q = 0$ for $n_j = 0$.

1.1.2 tangential variations

4. We seek to extremize the spectral width without changing the geometry of the interface. Accordingly, we restrict attention to tangential variations, i.e. variations of the form

$$\delta R = R_\theta \, \delta u, \tag{2}$$
$$\delta Z = Z_\theta \, \delta u. \tag{3}$$

5. To preserve stellarator symmetry, we consider $\delta u = \sum_k u_k \sin(m_k\theta - n_k\zeta)$.
6. The variations in the Fourier harmonics of R and Z are given by

$$\delta R_j = \oint\!\!\oint d\theta d\zeta \;\; R_\theta \, \delta u \, \cos(m_j\theta - n_j\zeta), \tag{4}$$
$$\delta Z_j = \oint\!\!\oint d\theta d\zeta \;\; Z_\theta \, \delta u \, \sin(m_j\theta - n_j\zeta), \tag{5}$$

7. The first variation in M as

$$\delta M = \oint\!\!\oint d\theta d\zeta \;\; (R_\theta \, X + Z_\theta \, Y)\, \delta u, \tag{6}$$

where $X = \sum_j (m_j^p + n_j^q) R_j \cos(m_j\theta - n_j\zeta)$ and $Y = \sum_j (m_j^p + n_j^q) Z_j \sin(m_j\theta - n_j\zeta)$

1.1.3 extremizing condition

8. The condition that $\delta M = 0$ for arbitrary δu is

$$I \equiv R_\theta \, X + Z_\theta \, Y = 0. \tag{7}$$

9. The derivatives of M with respect to the u_k are given

$$\frac{\partial M}{\partial u_k} = \oint\!\!\oint d\theta d\zeta \;\; (R_\theta X + Z_\theta Y)\sin(m_k\theta - n_k\zeta). \tag{8}$$

1.1.4 comments

10. For `pwidth`$=2$, and ignoring the n^q term, we see [1] that $X \equiv -R_{\theta\theta}$ and $Y \equiv -Z_{\theta\theta}$, and the extremizing condition reduces to $R_\theta R_{\theta\theta} + Z_\theta Z_{\theta\theta} = 0$, which is equivalent to the equal arc length condition, $R_\theta^2 + Z_\theta^2 = const$.

11. The derivatives of the spectral constraints, $I = R_\theta X + Z_\theta Y$, are derived using $\sin(\alpha+\beta) = [\sin(\alpha+\beta) + \sin(\alpha-\beta)]/2$ to give

$$
\begin{aligned}
\frac{\partial I}{\partial R_j} &= \frac{\partial R_\theta}{\partial R_j} X + R_\theta \frac{\partial X}{\partial R_j} \\
&= \frac{1}{2}\sum_k \left[-(m_j\lambda_k + m_k\lambda_j) R_k \sin(\alpha_j + \alpha_k) - (m_j\lambda_k - m_k\lambda_j) R_k \sin(\alpha_j - \alpha_k) \right] \qquad (9)\\
\frac{\partial I}{\partial Z_j} &= \frac{\partial Z_\theta}{\partial R_j} Y + Z_\theta \frac{\partial Y}{\partial R_j} \\
&= \frac{1}{2}\sum_k \left[+(m_j\lambda_k + m_k\lambda_j) Z_k \sin(\alpha_j + \alpha_k) - (m_j\lambda_k - m_k\lambda_j) Z_k \sin(\alpha_j - \alpha_k) \right]. \qquad (10)
\end{aligned}
$$

sw03aa.h last modified on 2012-12-18 ;

[1] S. P. Hirshman and J. Breslau. Explicit spectrally optimized fourier series for nested magnetic surfaces. *Phys. Plasmas*, 5(7), 1998.